Transverse Entanglement of Biphotons

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We measure the transverse entanglement of photon pairs on their propagation from the near to the far field of spontaneous parametric down-conversion. The Fedorov ratio, depending on the widths of conditional and unconditional intensity measurements, is shown to be only able to characterise entanglement in the near and far zones of the source. Therefore we also apply a different technique. Based on the interference of the two-photon amplitude, our measurement provides direct experimental access to the Schmidt number of the state. Our measurements show that the interferometric scheme always gives the full transverse entanglement of the quantum state.

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Entanglement is an exciting phenomenon and a fundamental resource in quantum information and quantum computation. One convenient source of entanglement are photon pairs (biphotons) obtained by spontaneous parametric downconversion (SPDC). These photons can be entangled not only in discrete variables like polarisation or photon number but also position and momentum, which are continuous [1–4]. The case of continuous variables is especially appealing for quantum informational tasks because it allows access to a larger Hilbert space [5].

Entanglement in the transverse wavevectors of SPDC biphotons can be understood in terms of the famous EPR scenario [6] where entanglement is identified by violation of the inequality

$$(\Delta x)^2 (\Delta p)^2 \geq \left(\frac{\hbar}{2}\right)^2, \tag{1}$$

where $(\Delta x)^2$ and $(\Delta p)^2$ are the variances of a quantum system in position and momentum respectively. The violation of Eq. (1) has been measured [8**?**] and even though it does quantify the amount of transverse entanglement in the biphoton state, measurement in both near- (position) and far field (momentum) of the source are necessary to obtain a value. Closely related to the violation of Equation (1) is the Fedorov ratio [9] which is especially appealing because it is an entanglement quantifier that can be directly measured. Unfortunately the Fedorov ratio does vary while the state propagates from the near to the far field region [10] and even turns to unity at some point, indicating no entanglement. Thus the Fedorov ratio cannot be considered a measure for the full entanglement at any arbitrary position. In this work, we demonstrate a measurement of the full evolution of the Fedorov ratio between those two regimes. Additionally we implement a different scheme which has been proposed [10] to fully quantify the transverse entanglement of the biphoton state. This measurement allows direct access to the Schmidt number [1, 11], a quantity usually unattainable in the laboratory but always giving the full amount of entanglement.

Consider the quantum state of a photon pair generated by SPDC at the distance z from the centre of the crystal as [12]

$$|\psi(z)\rangle = \int\int d\vec{p}\, d\vec{q}\, \Phi(\vec{p}, \vec{q}, z)\, \hat{a}_s^\dagger(\vec{p})\, \hat{a}_i^\dagger(\vec{q})|0\rangle \tag{2}$$

where $\vec{p}$ and $\vec{q}$ are the transverse wavevectors of the signal and idler photons and $\hat{a}_s^\dagger(\vec{p})$ and $\hat{a}_i^\dagger(\vec{q})$ their respective creation operators. The properties of the state are governed by the two photon amplitude $\Phi(\vec{p}, \vec{q}, z)$. The most widely used operational measure for the entanglement in such a system is the Fedorov ratio, which is given by

$$R = \frac{\Delta p}{\delta p} = \frac{\Delta q}{\delta q}. \tag{3}$$

Here Δp is the standard deviation of the marginal distribution of the two photon amplitude $P(\vec{p}, z) = \int d\vec{q}\, |\Phi(\vec{p}, \vec{q}, z)|^2$, which we will from here on refer to as the unconditional distribution. The width δp is given by the standard deviation of the conditional probability distribution $P(\vec{p}|\vec{q}, z) = |\Phi(\vec{p}, \vec{q}, z)|^2$ at a fixed value of $\vec{q}$. Analogous expressions hold for $\vec{p}$. Another important feature of the two photon amplitude is that both transverse dimensions are independent so that the two photon amplitude factorises: $\Phi(\vec{p}, \vec{q}) = \Phi(p_x, q_x)\, \Phi(p_y, q_y)$. This allows us to study the behaviour of the system utilising only the spatial degree of freedom in x-direction.

An intuitive approach to the Fedorov ratio is to think of entanglement as strong correlations between very uncertain occurrences. Thus it is clear that entanglement takes a high value for systems where the conditional distribution is very narrow (strong correlations) while the unconditional distribution is very broad (high uncertainty).

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The experimental setup is depicted on the right hand side of Fig. (1). In order to generate biphotons we focus a cw laser with a measured beamwaist of of 245μm onto a 2mm BBO crystal. By means of an interference filter of 8nm bandwidth and and apertures positioned at the the correct crystal angle, we select the degenerate non-collinear type I downconversion process: 404nm $\rightarrow$ 808nm. To select a certain part of the angular spectrum, we use slits of 30μm width and several mm height. The height of the slit leads to an integration over the y direction in both conditional and unconditional distributions which does not change the ratio between them. Diffraction from the slits is compensated by cylindrical lenses before the light is fibrecoupled into avalanche photodiodes. For the measurement one of the slits remains fixed at the maximum intensity of the angular spectrum while the other one is scanned along transverse direction (in Figure 1 x, perpendicular to the propagation direction z) by means of a linear translation stage. We utilise a coincidence electronic circuit to record the coincidence-rates as well as single count rates of both detectors as a function of the transverse displacement. A pair of lenses (f = 500mm) is employed to obtain the far field distribution of the angular photon spectra (which corresponds to the transverse momenta) in the focal plane. We apply gaussian fits to both the conditional and the unconditional distributions and use their respective variances to calculate the Fedorov ratio via Equation (3). A series of similar measurements is performed at various distances z from the source up to the image plane of the lenses, where the near field (where the coordinate is equivalent to the position quadrature) of the photon distribution is found. The three measurements at positions of particular interest are shown in Fig. 2. In the same figure one also finds the numerical simulations of the two-photon amplitudes in the position representation. It is evident that there are correlations in the near-field zone and anti-correlations in the far field zone while no correlations are observed at a certain distance from the crystal. The dependence of the Fedorov ratio on the distance is plotted in Fig. 3. Note, that because of the effect of the lenses, the far field is obtained at a smaller distance (focal length) than the near field (image plane). It can be clearly seen how the Fedorov ratio varies as the state propagates through space. In the far field we obtain a Fedorov ratio $R = 4.0 \pm 0.3$. As we move our detectors towards the near field, the Fedorov ratio decreases. We even observe a drop to $R = 1$ [10] at a certain distance from the crystal, which would indicate no entanglement in this position before the complete entanglement emerges again in the near field. We additionally compare our measurement results with a numerical simulation (Fig. 3) and find the curve in good agreement with our measured data. Since we observe that the amount of entanglement increases again after the drop and even is completely restored in the near field, the entanglement cannot be lost due to decoherence. So the question what happened to the 'lost' entanglement remains. The explanation, given by Chan *et. al.* [10], is that upon propagation, the entanglement of the state migrates from the wavefunction's modulus to its phase and back. Thus, since in the intermediate zone between near- and far field, the entanglement (or at least parts of it) resides in the phase of the state, it is inaccessible to intensity measurements such as the one we use to determine the Fedorov ratio. This prediction is clearly confirmed by our measurements. As shown by Tasca *et. al.* [13, 14], application of different fractional fourier transforms in signal and idler channels allows one to reveal intensity correlations in the intermediate zone. This is similar to the generalised quadrature measurements performed in [15, 16] and requires specific lens configurations for every distance z in both signal and idler channels. In the following we present a more general method, which works at arbitrary distances with the same configuration.

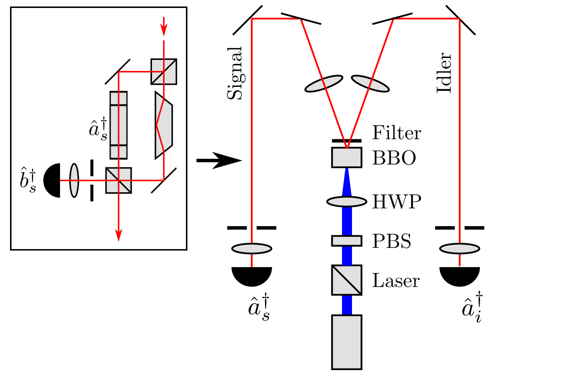
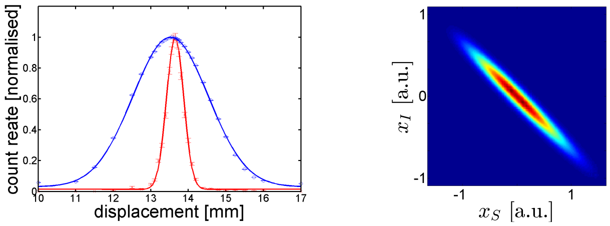
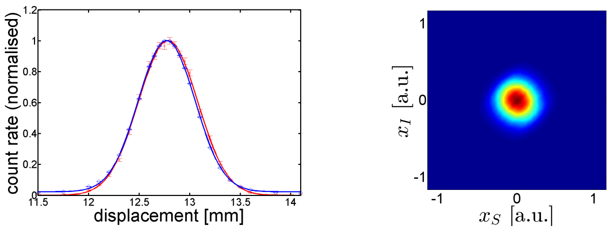


Figure 1. (Colour online) Experimental setup to determine the Fedorov ratio. Measurements are performed at different z positions by displacing the detectors. Inset on the left shows the modified Mach-Zehnder Interferometer we introduced in the signal arm, for the Schmidt number measurement.

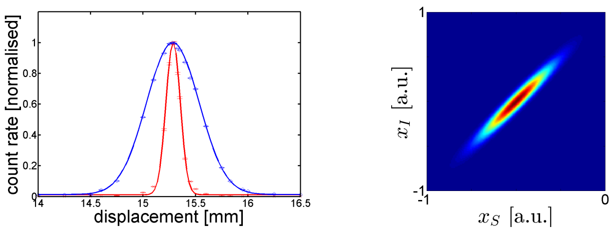
In this method, the measured quantity is the Schmidt number K, which is connected to the effective number of modes. Any pure state $|\Psi\rangle$ of a composite system AB can be expressed in a certain basis so that $|\Psi\rangle = \sum_i \sqrt{\lambda_i}|i_A\rangle|i_B\rangle$. The number of non-zero elements λ_i required to express the state vector in terms of the Schmidt basis $|i_A\rangle|i_B\rangle$ is directly related to the lack of separability of the state. For infinitely large Hilbert spaces the Schmidt number is defined as $K = 1/\sum_i \left(\lambda_i^2\right)$ [17]. Unlike the Fedorov ratio, the Schmidt number will always give the full amount of entanglement of a system by definition. The drawback however is that K is usually not considered an operational measure, meaning it cannot be easily obtained by a direct measurement. Nevertheless we were able to implement a setup which allows us experimental access to the Schmidt number. Since the entanglement can manifest itself also in the phase of the state, an interferometric setup was required. Unlike other schemes, our setup enables one to measure the full amount of entanglement without the need to perform measurements at specific positions, such as the near and far field [18]. Our scheme allows us to obtain the com-



(a)Far field ($z = 500$ mm)



(b)Intermedeate regime ($z = 1440$ mm)



(c)Near field ($z = 1550$ mm)

Figure 2. (Colour online) Measured conditional (red) and unconditional (blue) photon distributions with simulated two-photon amplitudes, 2(a) in the far field, 2(b) in the intermediate field where $R = 1$, and 2(c) in the near field.

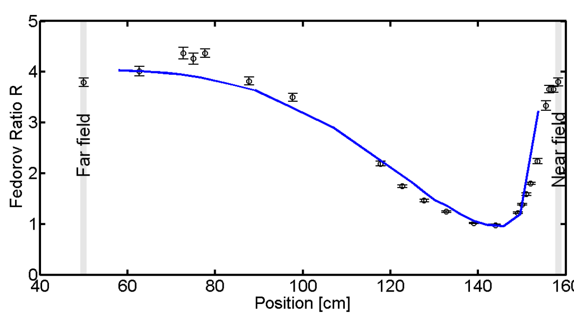


Figure 3. (Colour online) Measurement of the Fedorov ratio from far- to near field: Points show measurements, the solid line is a numerical simulation.

plete entanglement from just a single measurement in any arbitrary position. Following a proposal by Chan *et. al.* [10], we introduced a Mach-Zehnder like interferometer in the signal channel. A sketch can be found in the inset on the left of Fig. 1.The interferometer contains one dove prism in each arm, one of which is rotated by $\pi/2$ with respect to the other. Due to internal reflection within the prisms the beam in one arm of the interferometer is 'spatially mirrored' in one direction while the beam in the other arm is 'mirrored' orthogonally compared to the first. As this is equivalent to the inversion operation, one can say the beam is overlapped with its inverted copy on the final beamsplitter. For a single mode state, the whole crossection of the beam is able to interfere with itself and we would theoretically expect 100% of visibility between the signals at the outputs $\hat{a}_s^\dagger$ and $\hat{b}_s^\dagger$. With an increasing number of modes (which can be understood as an increase in the degree of entanglement) however, this visibility will drop as different modes will not interfere. A more rigorous mathematical treatment [10] shows that the Schmidt number is given by

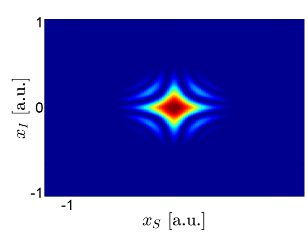
$$K = \frac{(P_+ + P_-)}{(P_+ - P_-)}, \tag{4}$$

where P_+ and P_- are the conditional count rates

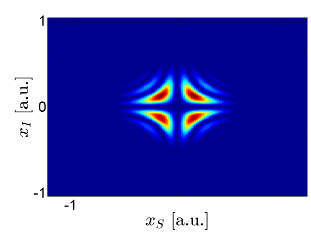
$$P_+ = \int\int dx_s dx_i P_{\hat{a}_s^\dagger \hat{a}_i^\dagger}(x_s, x_i) \tag{5}$$

$$P_- = \int\int dx_s dx_i P_{\hat{b}_s^\dagger \hat{a}_i^\dagger}(x_s, x_i)\,. \tag{6}$$

Here $P_{\hat{a}_s^\dagger \hat{a}_i^\dagger}(x_s, x_i)$ is the probability to observe the signal photon at the output $\hat{a}_s^\dagger$ of the interferometer at position x_s and the idler photon at position x_i in the idler mode $\hat{a}_i^\dagger$. Analogously $P_{\hat{b}_s^\dagger \hat{a}_i^\dagger}(x_s, x_i)$ describes the joint detection probability between the other interferometer output $\hat{b}_s^\dagger$ and idler $\hat{a}_i^\dagger$ at (x_s, x_i).
Figure 4 shows a numerical simulation of the two probability distributions at both interferometer outputs. According to Eq. (4), the Schmidt number is given by the inverse visibility of those coincidence probabilities integrated over x_s and x_i. Instead of using two detectors as



(a)Two photon intensity $P_{\hat{a}_s^\dagger \hat{a}_i^\dagger}(x_s, x_i)$



(b)Two photon intensity $P_{\hat{b}_s^\dagger \hat{a}_i^\dagger}(x_s, x_i)$

Figure 4. (Colour online) Simulation of the Two photon intensities at both outputs of the interferometer, 4(a) shows constructive interference, 4(b) depicts the complementary destructive interference between the interferometer arms.

indicated in Fig. 1 we measured the visibility in coincidences by scanning the phase of the interferometer with the help of a piezo attached to one of the mirrors. This time we position two $200\mu m$ slits oriented along the x direction in both signal and idler arms. This is equivalent

to an integration along x_s and x_s in one single measurement. Hence we are able to determine the Schmidt number simply by moving the piezo from the position of maximum count rate to minimum count rate in one output of the interferometer and record the visibility. Both the results of the measurements as well as the numerical predictions are depicted in Fig. 5. Please note that the numerical model does not perform the actual Schmidt decomposition of the quantum state but rather simulates the effect of our measurement method on the state. It is apparent that the measurement points exclusively lie above the numerical prediction, which is not surprising because the entanglement we are trying to determine actually manifests itself in the lack of visibility. Accordingly any experimental imperfection always will lead to an overestimation of entanglement and consequently a slightly increased Schmidt number.

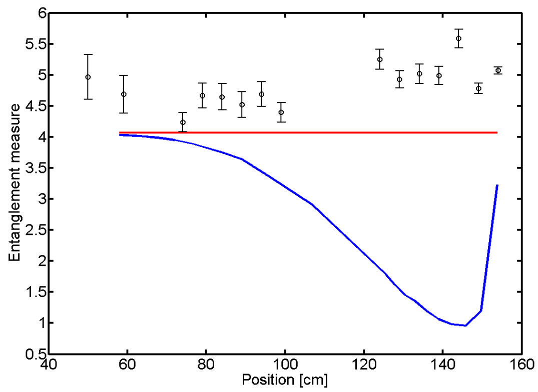


Figure 5. (Colour online) Circles are measurements of the Schmidt number, the solid red line is the result of a numerical simulation of the Schmidt number measurement. For comparison the numerical simulation of the Fedorov ratio is included as well (solid blue line).

As mentioned before, the reason for the breakdown of R is that upon propagation in free space the two-photon amplitude acquires a quadratic phase

$$\Phi(q,z) = \Phi(q,0)\exp(\imath kz)\exp\left(-\imath\frac{q^2}{2k}z\right), \qquad (7)$$

where q is again the transverse component of the wavevector k. Henceforth, this leads to a shift of entanglement from the modulus to the phase of the quantum state at a certain position. The quadratic phase in Eq. (7) is mathematically identical to the one acquired by the two-photon amplitude due to the group velocity dispersion in an optical fibre [19]. Thus it should be possible to obtain similar results in the time-frequency entanglement of spontaneous parametric downconversion photons using a fibre as dispersive medium. Such effects could be an issue in QKD experiments where two-photon states are sent through very long fibres.

In conclusion we observed the migration of entanglement in the transverse momentum of the quantum state emitted by spontaneous parametric downconversion from modulus to phase as the photons propagate through free space. We present both numerical and experimental results showing the seemingly vanishing entanglement between near and far field of the crystal as given by the Fedorov ratio. Furthermore we implement a direct measurement of the Schmidt number, which unambiguously gives the full entanglement of the quantum state. Finally we propose an analogue experiment which could reveal similar behaviour in the time-frequency picture.

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